Компьютерное восстановление параметров флюидонасыщенности и горной породы (кварц-полевошпатовый коллектор) по данным измерений ИНГК (неупругого рассеяния).

А.И. Хисамутдинов, ИНГГ СО РАН

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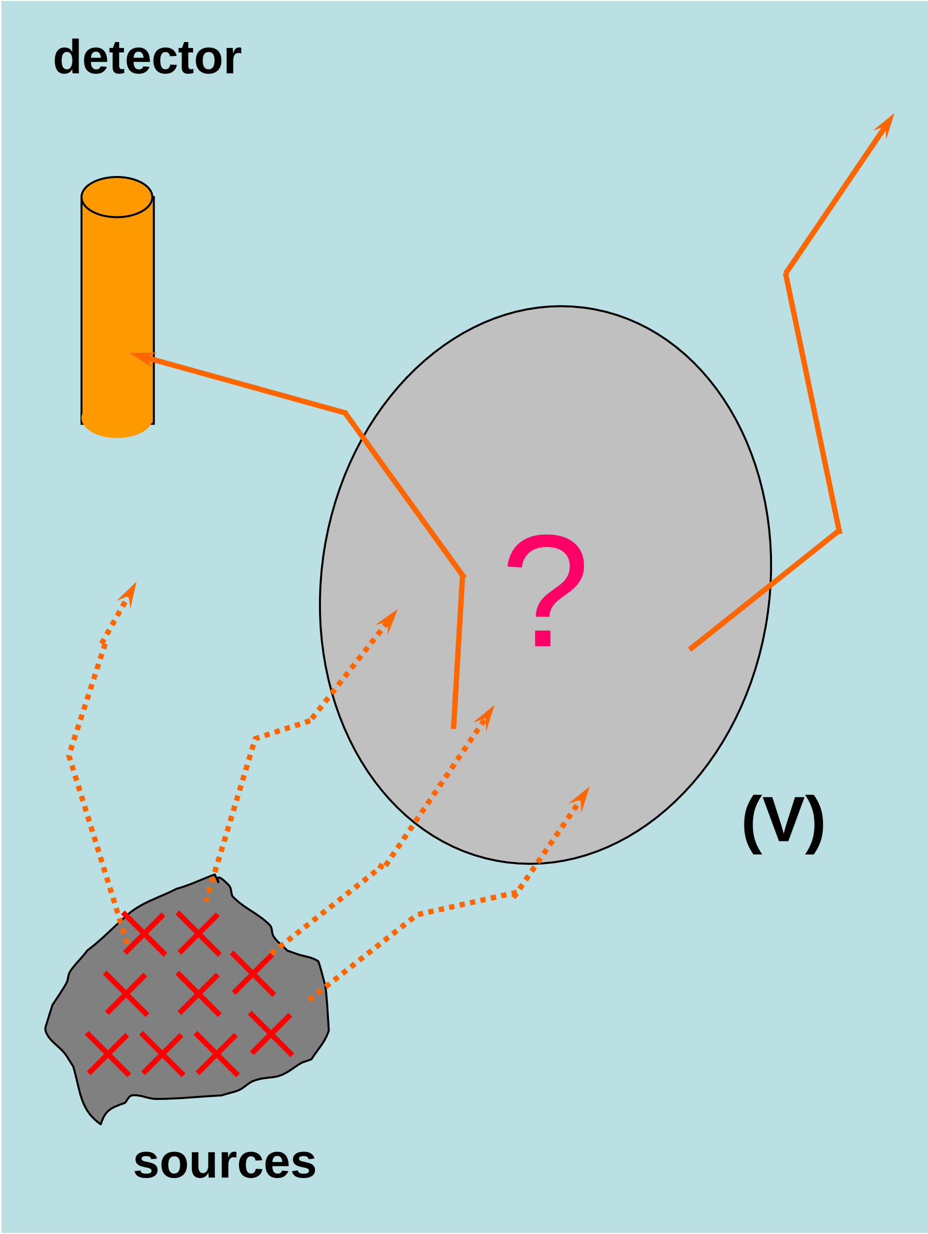
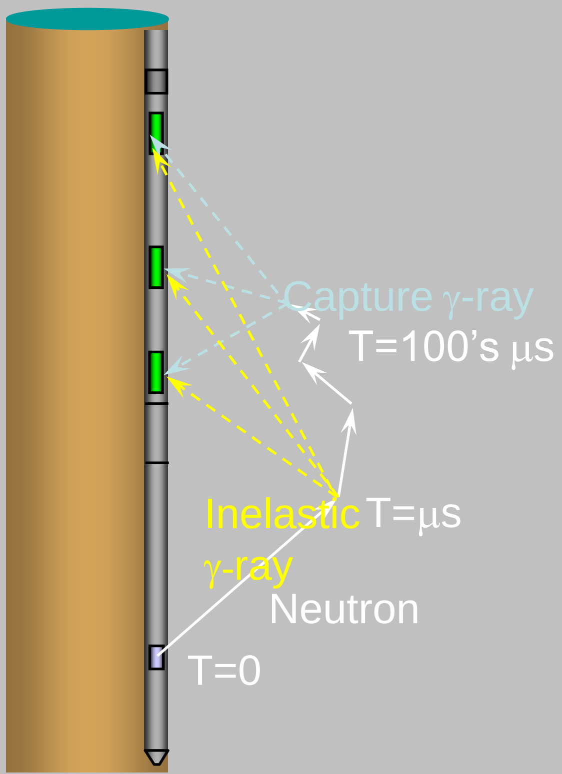

detector
?
(V)
sources



Capture γ-rays:
Hydrogen: 2.23 MeV
Iron: 7.65 MeV
Chlorine: 6.11, 1.95 MeV
Calcium: 6.41, 1.96 MeV
Silicon: 4.95, 3.54 MeV

Inelastic γ-rays:
Carbon: 4.44 MeV
Oxygen: 6.13 MeV
Silicon: 1.78 MeV
Calcium: 3.34 MeV

Notations, model of medium, cross-sections

$$(X) = R^3 \bigotimes (S_1) \bigotimes [0, \infty)$$
$$x \equiv (r, \Omega, E), \; v = v(E), \; v = |v|\Omega$$
$$(G) = (V_G) \bigotimes (S_1) \bigotimes [0, \infty), \; (G) \subset (X)$$

$$\begin{cases} \frac{\partial \phi_{in}}{\partial t} + (v, \nabla \phi_{in}) + \Sigma_{in} \Phi_{in} = \hat{S}^{+}_{in} \Phi_{in} + q_{in}, \\ \frac{\partial \phi}{\partial t} + (v, \nabla \phi) + \Sigma \Phi = \hat{S}^{+} \Phi + \hat{Q}^{+} \Phi_{in} \end{cases}$$

- first problem

$$\frac{\partial \phi}{\partial t} + (v, \nabla \phi) + \Sigma \Phi = \hat{S}^{+} \Phi + q_0$$

- second problem

$\phi_{in}(t, x), \; \phi(t, x)$ **- phase densities**

$$\Phi_{in}(t, x) \equiv |v| \phi_{in}(t, x), \; \Phi(t, x) \equiv |v| \phi(t, x)$$

$$S_{in}(\Omega, E \to \Omega', E' \,|\, r), S(\Omega, E \to \Omega', E' \,|\, r), Q(\Omega, E \to \Omega', E' \,|\, r)$$
$$\left[\hat{S}_{in} \cdot 1\right](x) \le \Sigma_{in}(x), \left[\hat{S} \cdot 1\right](x) \le \Sigma(x), \left[\hat{Q} \cdot 1\right](x) < C_Q < \infty$$

Integral form of equations:

$$\left\{ \begin{array}{l} \phi_{in} = \hat{\mathcal{T}}_{in}^{+} \hat{S}_{in}^{+} \Phi_{in} + \hat{\mathcal{T}}_{in}^{+} q_{in} \\ \phi = \hat{\mathcal{T}}^{+} \hat{S}^{+} \Phi + \hat{\mathcal{T}}^{+} \hat{Q}^{+} \Phi_{in}, \end{array} \right.$$

$$\phi = \hat{\mathcal{T}}^{+} \hat{S}^{+} \Phi + \hat{\mathcal{T}}^{+} q_0,$$

$n_d = 1$

Model of composition of (V)

$\Lambda\alpha = \bar{\lambda}$ **- system of (K-N) constraint equations**

where Λ - $(K-N)$ **matrix** $(K-N)\times K$

$\alpha = (\alpha_1, \cdots, \alpha_K)^T$ **- volume parts.** $\bar{\lambda}$ **- column (K- N) in height**

$\Lambda_{neq}\alpha < \lambda_{neq}$ **- System** of K_{neq} **inequalities, where** Λ_{neq} and λ_{neq} -

($K_{neq}\times$**K) matrix** and **column**

$$\mathcal{A} \equiv \{\alpha : \alpha \in R^K, \quad \Lambda_{neq}\alpha < \lambda_{neq}, \quad \Lambda\alpha = \lambda\}, \quad \mathcal{A} \subset R^K$$

$$\forall k \in \overline{1, K} : 0 < \alpha_k < 1.$$

$\rho_j, \quad j = \overline{1, N_e}$ **- numerical densities of all elements**

$$[1, \cdots, N], \quad [(N+1), \cdots, N_e]$$

$\rho = (\rho_1, \cdots, \rho_N)^T$ - Densities of **characteristic elements**

$\rho = \Upsilon\alpha$, **where** $\Upsilon - N \times K$ **matrix,** $\alpha \in \mathcal{A}$

$$\mathcal{U} \equiv \{\rho : \rho = \Upsilon\alpha, \quad \alpha \in \mathcal{A}\}$$

$\rho = \Upsilon\alpha, \quad \alpha \in \mathcal{A}, \quad \rho \in \mathcal{U}$ **- one-to-one mapping** !

$$\Upsilon_M : \mathcal{A} \to \mathcal{U}, \quad \rho = \Upsilon_M(\alpha) = \Upsilon\alpha \quad \forall\alpha \in \mathcal{A}; \quad \mathcal{U} = \Upsilon_M(\mathcal{A})$$

$\Upsilon_M^{-1} : \mathcal{U} \to \mathcal{A}, \quad \alpha = \Upsilon_M^{-1}(\rho)$. $\mathrm{B}\alpha = (\rho;\lambda)$, $(\rho;\lambda)^T = (\rho_1,...,\rho_N;\lambda_1,...,\lambda_{K-N})$, **matrix** B **is composed from** Υ **and** Λ **.**

$n_d > 1$

$$(V_i) \subset (V)$$

$$K = \sum_{i=1}^{n_d} K_i, \quad N = \sum_{i=1}^{n_d} N_i; \quad \mathcal{A} \equiv \mathcal{A}_1 \otimes \cdots \otimes \mathcal{A}_{n_d}, \quad \mathcal{U} \equiv \mathcal{U}_1 \otimes \cdots \otimes \mathcal{U}_{n_d}$$

$$K_{neq} = \sum_{i=1}^{n_d} K_{neq,i};$$

$$\alpha \equiv (\alpha_1^1, \cdots, \alpha_{K_1}^1, \cdots, \alpha_1^{n_d}, \cdots, \alpha_{K_{n_d}}^{n_d})^T, \ \alpha \subset \mathcal{A}$$

$$\rho \equiv (\rho_1^1, \cdots, \rho_{N_1}^1, \cdots, \rho_1^{n_d}, \cdots, \rho_{N_{n_d}}^{n_d})^T, \ \rho \subset \mathcal{U}$$

In problem 1 $\forall x \in (G_V)$:

$$\hat{Q}^+ = \hat{Q}_0^+ + \hat{Q}_b^+$$

$$\hat{Q}_b^+ = \sum_{j=1}^{N} \rho_j \hat{q}_j^+ \quad \forall x \in (G_V)$$

In problem **2 :** $\hat{S}^+ = \hat{S}_0^+ + \hat{S}_b^+$

$\hat{S}_b^+ \equiv \hat{O} \quad \forall x \notin (G_V)$ **and** $\hat{S}_b^+ = \sum_{j=1}^{N} \rho_j \hat{b}_j^+ \quad \forall x \in (G_V)$

in (V): $\hat{S}_0^+ = \sum_{j=1}^{N_e} \rho_j \hat{s}_j^+$

$$\Sigma(x) = \sum_{j=1}^{N_e} \rho_j \sigma_j(x)$$

$$\Sigma_{in}(x) = \sum_{j=1}^{N_e} \rho_j \sigma_{in,j}(x), \quad \hat{S}^+ = \sum_{j=1}^{N_e} \rho_j \hat{s}_j^+, \quad \hat{S}_{in}^+ = \sum_{j=1}^{N_e} \rho_j \hat{s}_{in,j}^+$$

$\hat{S}_b^+, \ \hat{Q}_b^+$ **!** **Characteristic** interactions

Statement of inverse problems-1

$I_i(\alpha) = \iint \Phi(t,x/\alpha) \mathrm{E}_i(t,x) dt dx$ - **linear functional on phase density**

i=1,…,N_M

$I(\alpha) = (I_1(\alpha),\ldots,I_{N_M}(\alpha))^T$ **- Column N_M in height**

$d = (d_1,\ldots,d_{N_M})^T$ **- data column**

$\forall \alpha \in \mathcal{A}$ **supposed solutions of both problems exist and unique($\exists$!)**

$$I(\alpha) = d \qquad \textbf{(3.1)}$$

$$I : \mathcal{A} \rightarrow \mathcal{I}, \quad \mathcal{I} \subset R^{N_M} \qquad \textbf{(3.2)}$$

considered mapping (3.2) is continuous and bounded

$$I \circ \Upsilon_M^{-1} : \mathcal{U} \rightarrow \mathcal{I} . \qquad \textbf{(3.3)}$$

We state inverse problems for both cases as problems subsequent upon (3.1) under conditions:

i. unknowns α are in $\mathcal{A}_1$, $\mathcal{A}_1 \subset \mathcal{A}$

ii. Contraction of mapping (3.2) to $\mathcal{A}_1$ is homeomorphic mapping

iii. d $\in$ $\mathcal{I}(\mathcal{A}_1)$

denote $\mathcal{U}_1 \equiv \Upsilon_M(\mathcal{A}_1)$

Superposition principle.

$$[\hat{Q}_0^+ \Phi_{in}](\cdot) + \sum_{j=1}^{N} \rho_j [\hat{q}_{bj}^+ \Phi_{in}](\cdot)$$

$$\phi(\cdot) = \phi_0(\cdot) + \sum_{j=1}^{N} \phi_{bj}(\cdot) \qquad \textbf{(3.5)}$$

$$I_i = I_i^{(0)} + \sum_{j=1}^{N} I_{ij}, \quad i = \overline{1, N_M} \qquad \textbf{(3.6)}$$

$$\begin{cases} \dfrac{\partial \phi_0}{\partial t} + (v, \nabla \phi_0) + \Sigma \Phi_0 = \hat{S}_0^+ \Phi_0 + q_0 \\ \dfrac{\partial \phi_{bi}}{\partial t} + (v, \nabla \phi_{bi}) + \Sigma \Phi_{bi} = \hat{S}_0^+ \Phi_{bi} + q_i \\ \Phi_0 = |v| \phi_0, \quad \Phi_{bi} = |v| \phi_{bi}, \quad q_i(\cdot) = \rho_i [\hat{b}_i^+ \Phi](\cdot), \quad i = \overline{1, N} \end{cases}$$

$$I_{ij}/\rho_j = O(1), \; \rho_j \to 0, \quad i = \overline{1, N_M}, j = \overline{1, N} \qquad \textbf{(3.8)}$$

Proposition 3.1. For both problems (3.6) and (3.8) take place

$I^{(0)} \equiv (I_1^{(0)}, \cdots, I_{N_M}^{(0)})^T$, $A \equiv ||a_{ij}||$ - ($N_M \times N$) matrix with elements $a_{ij}(\alpha) = I_{ij}(\alpha)/\rho_j$.

$c(\alpha) \equiv d - I^{(0)}(\alpha),\ \alpha \in \mathcal{A}$

matrix of characteristic interactions

$$I^{(0)} + A\rho = d$$

Statement of inverse problems-2, starting from 3.1

1) $$A(\alpha)\rho - c(\alpha) = 0\,,$$

$$\rho - \Upsilon\alpha = 0, \qquad (\alpha, \rho) \in \mathcal{A}_1 \otimes \mathcal{U}_1,$$

$$Rg\,A(\alpha) = N \quad \forall \alpha \in \mathcal{A}_1$$

$$Rg\,A \circ \Upsilon_M^{-1}(\rho) = N \quad \forall \rho \in \mathcal{U}_1$$

2) $$A(\alpha)\Upsilon\alpha - c(\alpha) = 0, \quad \alpha \in \mathcal{A}_1,$$

$$A \circ \Upsilon_M^{-1}(\rho) \equiv A(\Upsilon_M^{-1}(\rho)), \quad c \circ \Upsilon_M^{-1}(\rho) \equiv c(\Upsilon_M^{-1}(\rho)).$$

3) $$A \circ \Upsilon_M^{-1}(\rho)\rho - c \circ \Upsilon_M^{-1}(\rho) = 0, \quad \rho \in \mathcal{U}_1$$

Definition of iteration processes

1) $$\inf\left((A\rho - c)^T, W(A\rho - c)\right), \quad (\alpha, \rho) \in \mathcal{A}_1 \otimes \mathcal{U}_1,$$ under condition $\rho = \Upsilon\alpha$

W – diagonal weight matrix

3) $$\inf\left((A \circ \Upsilon_M^{-1}(\rho)\rho - c \circ \Upsilon_M^{-1}(\rho))^T, W(A \circ \Upsilon_M^{-1}(\rho)\rho - c \circ \Upsilon_M^{-1}(\rho))\right)$$
$$\rho \in \mathcal{U}_1$$

Successive approximations by characteristic interactions for solving SEPE, using quadratic program approach

$$A(\alpha_{|s}) = A \circ \Upsilon_M^{-1}(\rho_{|s}) \equiv A_{|s}, \quad c(\alpha_{|s}) = c \circ \Upsilon_M^{-1}(\rho_{|s}) \equiv c_{|s}$$

for 3) $$\{\rho_{|s}\}_{s=0}^{\infty}: \min\left((A_{|s-1}\rho'_{|s} - c_{|s-1})^T, W(A_{|s-1}\rho'_{|s} - c_{|s-1})\right), \quad \rho'_{|s} \in \bar{\mathcal{U}},$$
$$s = 1, 2, \cdots; \quad \rho_{|0} \in \mathcal{U};$$

$\bar{\mathcal{U}}$ - convex closed bounded domain, $\partial\mathcal{U}$ - the bound of domain $\mathcal{U}$

Set $\rho_{|s} = \rho'_{|s}$, if $\rho'_{|s} \in \mathcal{U}$; otherwise, if $\rho'_{|s} \in \partial\mathcal{U}$, then set $\rho_{|s}$ equal to some point in $\mathcal{U}$ close to the $\rho'_{|s}$

for 1) $\{(\rho_{|s}, \alpha_{|s})\}_{s=0}^{\infty}$:

$$\min\left((A_{|s-1}\rho' - c_{|s-1})^T, W(A_{|s-1}\rho' - c_{|s-1})\right), \quad \rho' \in \overline{\Pi'_U}, \quad \alpha_{|0} \in \mathcal{A};$$

$$\min\left((B\alpha' - (\rho_{|s};\lambda))^T, (B\alpha' - (\rho_{|s};\lambda))\right), \quad \alpha' \in \overline{\mathcal{A}}$$

Details are analogously to the previous case 3)

Convergence of the iteration process for 3)

Assumptions: the iteration process is within $\mathcal{U}_1$, $N_M = N$

$$\rho_{|s} = (A_{|s-1})^{-1} c_{|s-1}, \qquad 1 \le s, \quad \rho_{|s} \in \mathcal{U}_1$$

$$\varphi(\rho) \equiv A^{-1} c(\Upsilon_M^{-1}(\rho)). \qquad \varphi : \mathcal{U}_1 \to R^N$$

$$\rho = \varphi(\rho), \quad \rho \in \mathcal{U}_1$$

$$\rho_{|s} = \varphi(\rho_{|s-1}), \quad \forall s \quad \rho_{|s} \in \mathcal{U}_1$$

$$\Delta_{|ik}(\rho) = -\sum_{l=1}^{N} \frac{\partial a_{il}}{\partial \rho_k}\rho_l + \frac{\partial c_i}{\partial \rho_k},$$

$$\tilde{\Delta}_{|ik}(\rho) = -\sum_{l=1}^{N} \frac{\partial a_{il}}{\partial \rho_k}\varphi_l(\rho) + \frac{\partial c_i}{\partial \rho_k}$$

Lemma 4.1 $\quad \varphi' = A^{-1}\tilde{\Delta}$

Proposition 4.1. **In** U **,** $U \subset \mathcal{U}_1$ **and ρ*** $\in U$**, let the following inequality** $\|A^{-1}\Delta\| < 1$ **be true**

Then a domain U^* **exists:** $U^* \subset \mathcal{U}_1, \ \rho^* \in U^*$

$$\varphi(U^*) \subseteq U^*, \quad \sup_{U^*} \|A^{-1}\tilde{\Delta}\| < 1$$

For problem 1:

$$\Delta_{|ij} = (\Phi_{in}, \frac{\partial \Sigma_{in}}{\partial \rho_j} \psi^{+}_{in|i}) - (\Phi_{in}, \frac{\partial \hat{S}_{in}}{\partial \rho_j} \psi^{+}_{in|i}) + (\Phi, \frac{\partial \Sigma}{\partial \rho_j} \psi^{+}_{i}) - (\Phi, \frac{\partial \hat{S}}{\partial \rho_j} \psi^{+}_{i})$$

For problem 2:

$$\Delta_{|ij} = +(\Phi, \frac{\partial \Sigma}{\partial \rho_j} \psi^{+}_{i}) - (\Phi, \frac{\partial \hat{S}_0}{\partial \rho_j} \psi^{+}_{i})$$

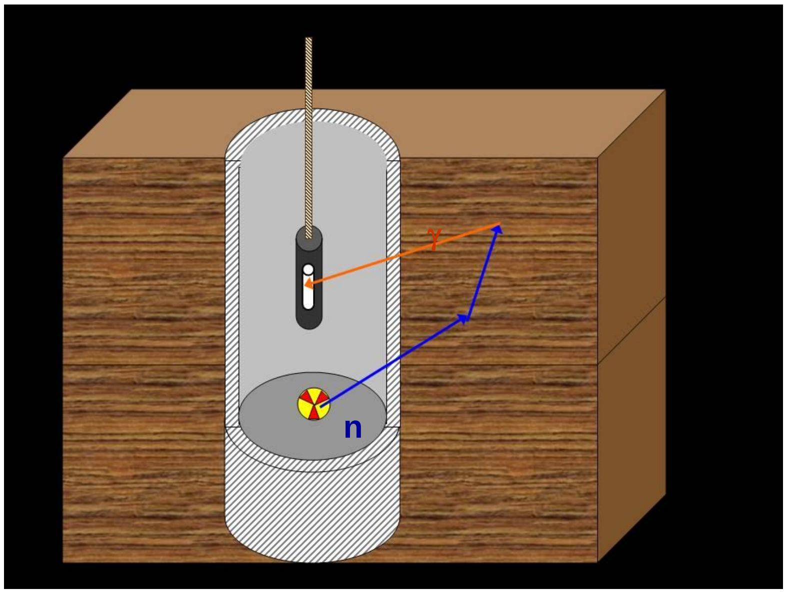
$$I' = A - \Delta, \quad \|A^{-1}\Delta\| < 1$$

The characteristic interactions should exert most influence on the derivatives

$$\psi^{+}_{.} = \hat{T}|v|\hat{S}\psi^{+}_{.} + \hat{T}|v|\mathcal{E}_{.} \qquad \psi^{+}_{in,.} = \hat{T}_{in}|v|\hat{S}_{in}\psi^{+}_{.} + \hat{T}_{in}|v|\hat{Q}\psi^{+}_{.}$$

Formation composition evaluation using data of pulsed neutron-gamma inelastic

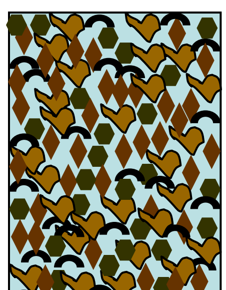
2 layers: borehole, H_2O — formation minerals + fluids

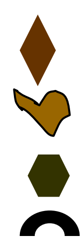


C, O, Ca, Si
{ρ1, ρ2, ρ3, ρ4}

$$\alpha_{H2O} + \alpha_{CH2} = kp$$

$$\alpha_{SiO2} + \alpha_{CaFS} + \alpha_{KFS} + \alpha_{NaFS} = 1 - kp$$



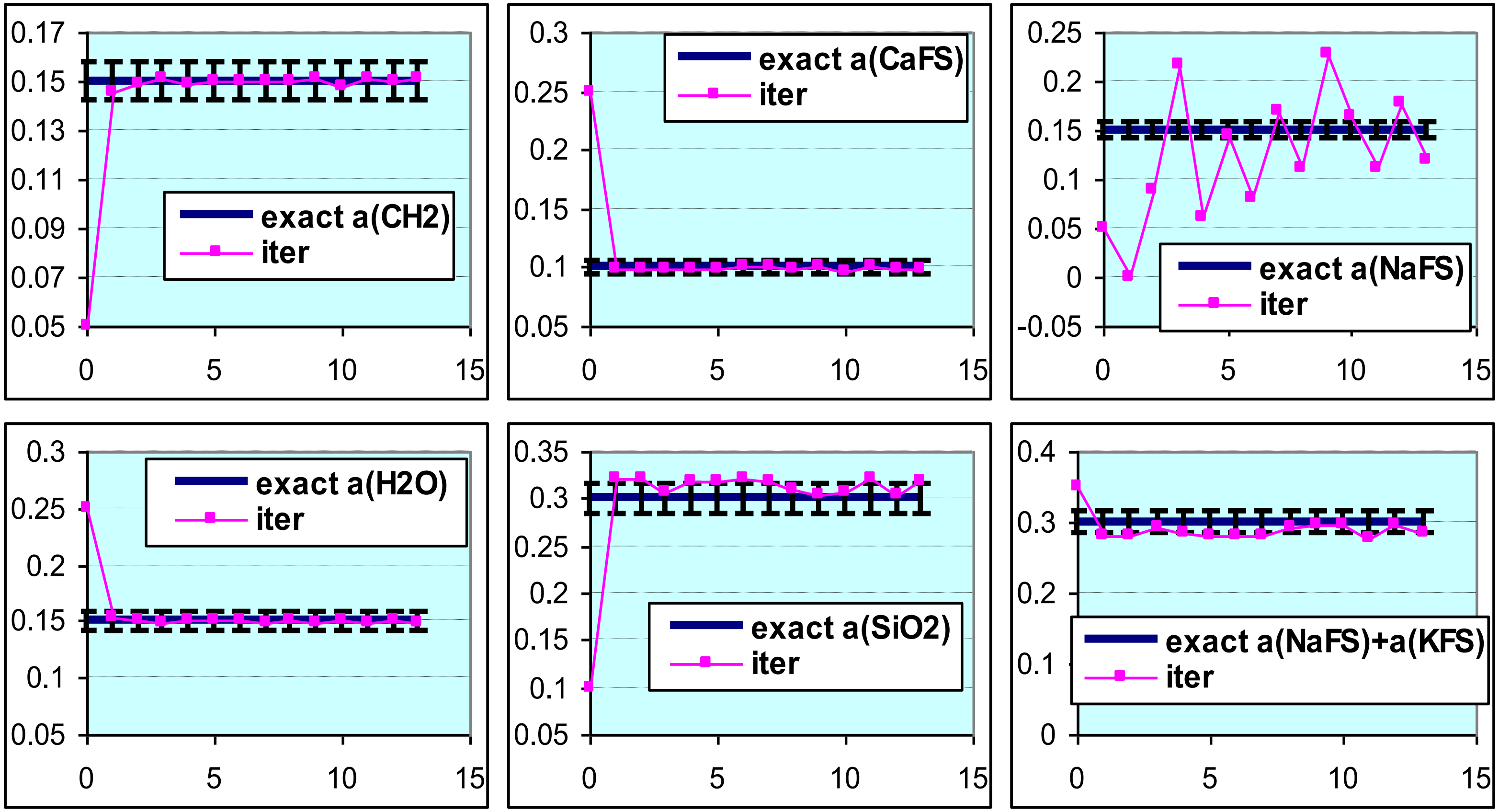


Compound	*formula*	
Water	H_2O	(kp)
Oil	CH_2	
Quartz	SiO_2	
Anorthite	$CaAl_2Si_2O_8$ (CaFS)	(1-kp)
Orthoclase	$KAlSi_3O_8$ (KFS)	
Albite	$NaAlSi_3O_8$ (NaFS)	

symmetrical medium consisting of two layers; first layer is fresh water (wellbore, radius <9.85cm) and second one is formation (radius >=9.85cm). The formation was described by the model of the medium given above where bulk fractions of all components were variable. At the axis of the wellbore a pulsed (d(t)) monoenergetic (d(E-14.1 MeV)) neutron source was placed, and at a distance Z=30cm a gamma-ray detector was located, its detection function *ei*(*t,x*)=1 in the corresponding i-th energy interval, i=1,…,4. Energies of gamma quanta caused by inelastic scattering were the

following: C – 4.44 MeV; O – 6.13 MeV; Ca – 3.34 MeV; Si – 1.77 MeV.

Convergence of the iteration process



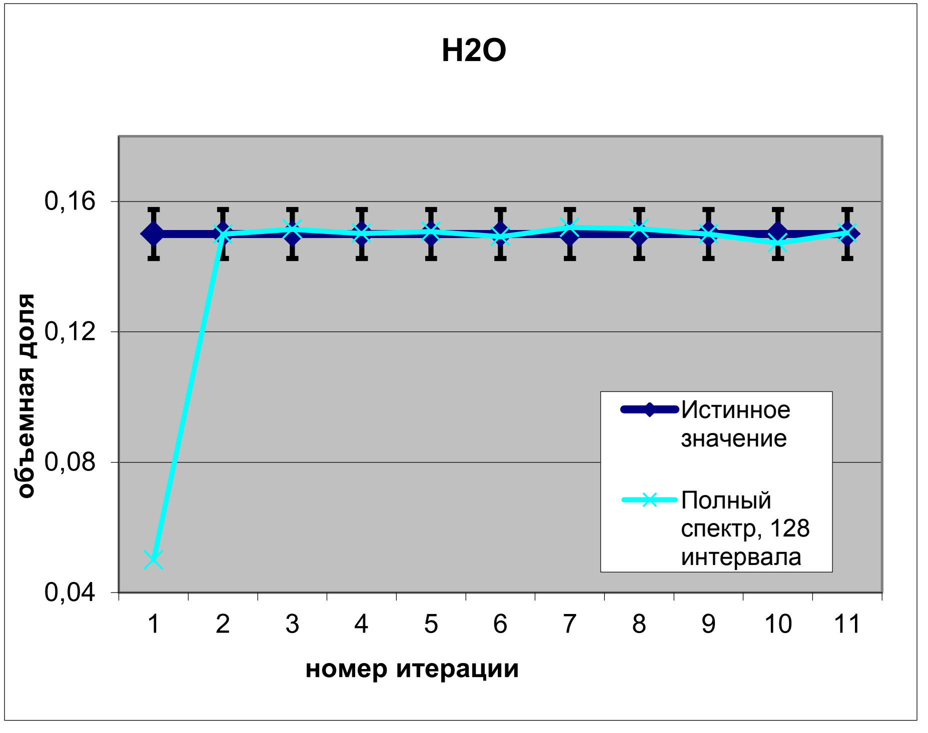
example: α(CH2)=0.15, α(H2O)=0.15 (*kp*=0.3), α(SiO2)=0.30, α(CaFS)=0.1, α(NaFS)=0.15, α(KFS)=0.15.

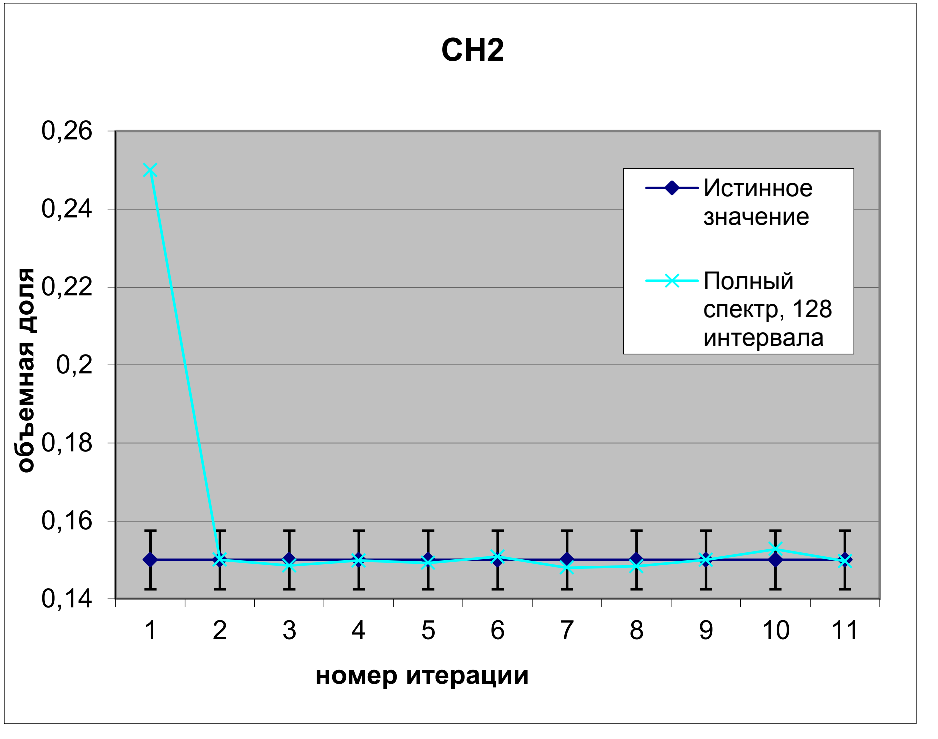
Осесимметричная среда состоит из двух цилиндрических слоёв: скважины радиуса 9.85 см и горной породы; первый заполнен водой, в качестве горной породы рассматривается кварц-полевошпатовый коллектор. Параметры последнего те же, что и на слайде 15. Плотность источника есть

$$q(t,x) = \frac{1}{2\pi}\delta(t)\delta(E - E_0)\delta(\rho - \rho_0)\delta(z)\delta(\mu - \mu_0)\delta(\vartheta - \phi) \ .$$

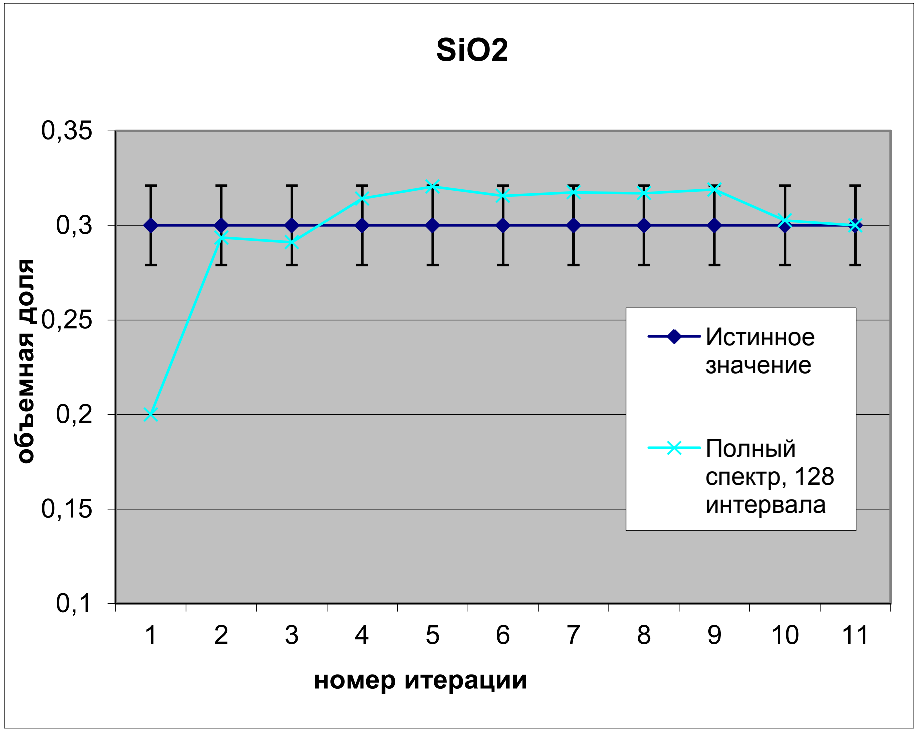
E_0 = 14.1 MeV. Вычислялись средние потоки гамма-квантов в цилиндре радиуса 7.5 см , высоты 15 см и с центром на оси скважины на расстоянии 40 см от центра источника в соответствующих энергетических интервалах. Последние располагались между 0 MeV и 6.13 MeV .

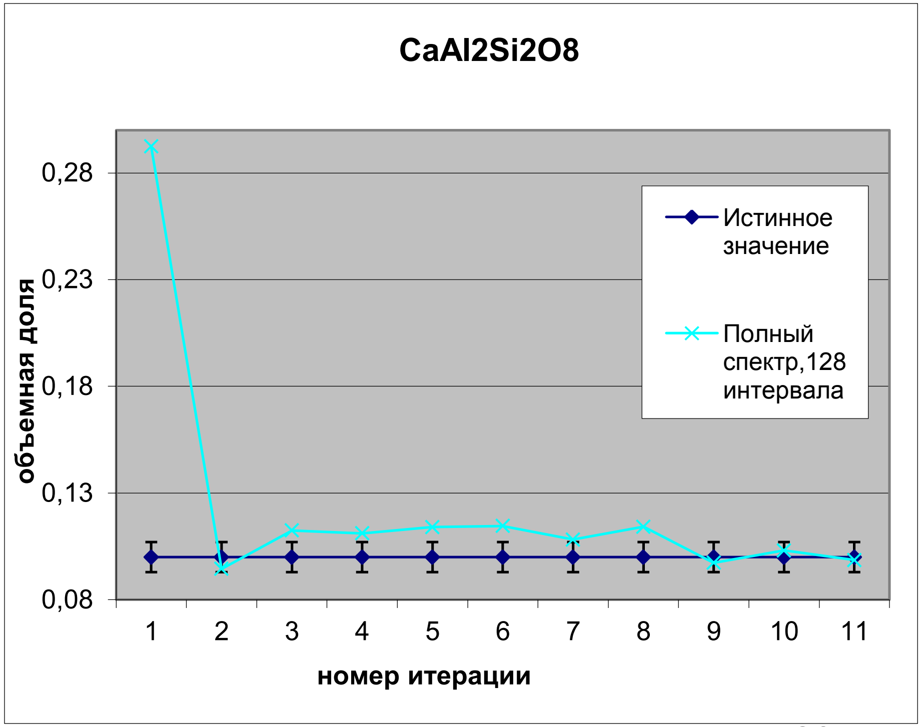
Convergence of the iteration process



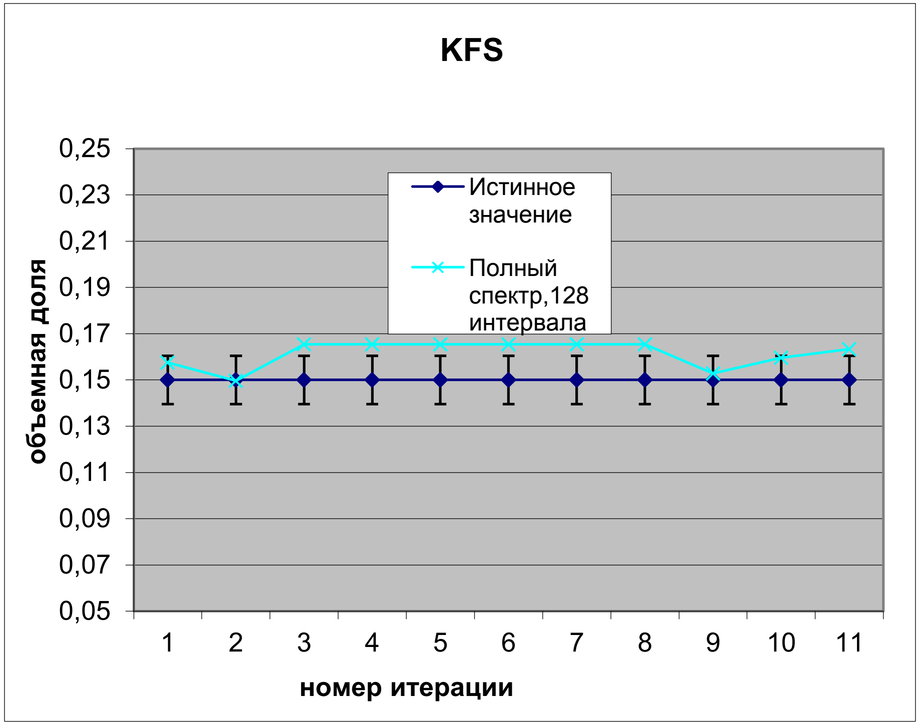


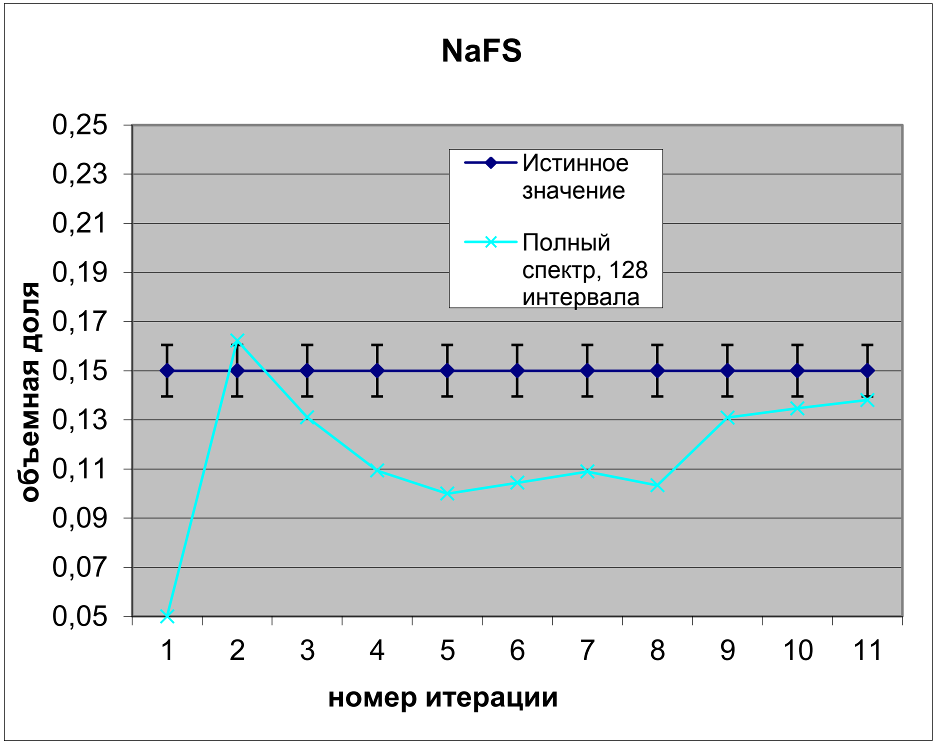
Convergence of the iteration process



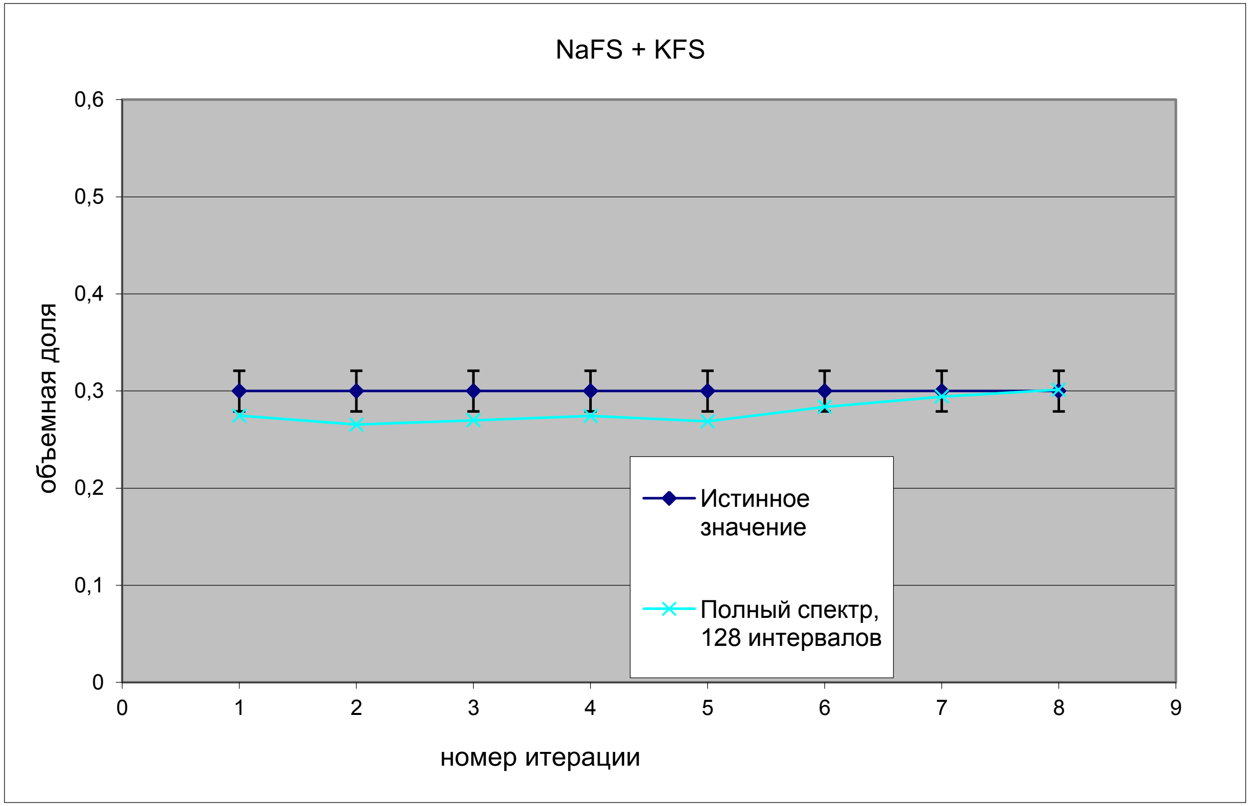


Convergence of the iteration process





Convergence of the iteration process



Neutron Activation Logging (in [2-3])

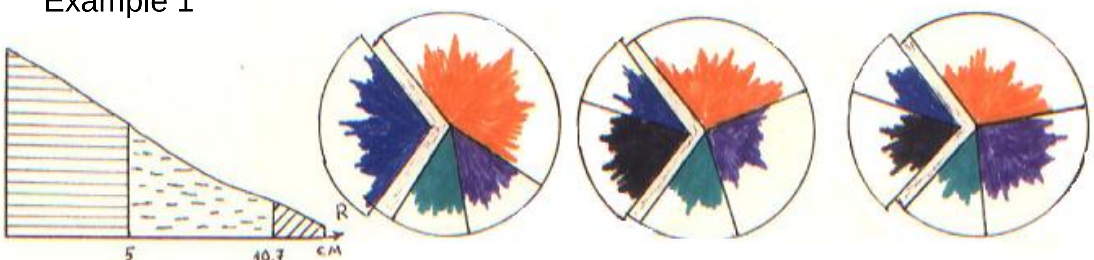
Result of numerical experiment in open and cased borehole

- device
- water
- steel pipe
- cement
- rock

- water
- oil
- plagioclase
- orthoclase
- quartz

Water – α_1
Oil – α_2
Plagioclase – α_3
Orthoclase – α_4
Quartz – α_5

Example 1



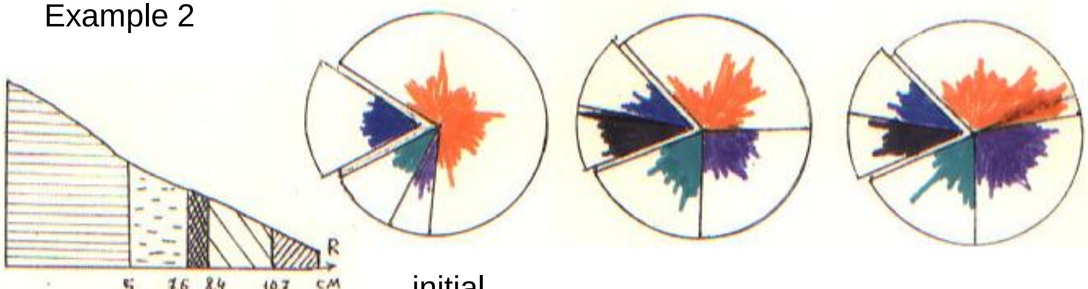
initial approximation | exact solution | 5-th iteration

Porosity k_p is considered to be given

$$\alpha_1 + \alpha_2 = k_p$$

$$\alpha_3 + \alpha_4 + \alpha_5 = 1 - k_p$$

Example 2



initial approximation | exact solution | 3-d iteration

O	Si	Al
ρ_1	ρ_2	ρ_3

Illustration

$$OZ, \quad X = R \otimes \{-1, 1\}, \, x = (r_3, \mu) \in X$$

$$(V) = (0, H), \;\; 0 \, < \, H \, < \, +\infty$$

$$q_0(x) \, = \, \delta(r_3) \cdot \delta_{\mu,1}$$

$$I \, = \, \Phi(0, -1)$$

$$\Sigma(x) = \rho\sigma_0, \, S_b(\mu' \to \mu|r_3) = (1/2)c\rho\sigma_0, \;\; S_0(\mu' \to \mu|r_3) = 0$$

$$I(\rho) \, = \, \frac{(1/2)\Sigma_s \cdot \mathrm{sh}(\tilde{\Sigma}H)}{(\Sigma_a + \Sigma_s/2)\,\mathrm{sh}(\tilde{\Sigma}H) + \tilde{\Sigma}\,\mathrm{ch}(\tilde{\Sigma}H)}$$

$$\Sigma_s = c\rho\sigma_0, \;\; \Sigma_a = (1-c)\rho\sigma_0, \; \tilde{\Sigma}^2 = \Sigma_a\Sigma = (1-c)\rho^2\sigma_0^2$$

$$z = \tilde{\Sigma}H = \rho\sqrt{(}1{-}c)\sigma_0 H, 0 < z < z_{max} < +\infty$$

$$z_{max} = \rho_{max}\sqrt{(1-c)}\sigma_0 H$$

$$I(\rho) = J(z) = \frac{1}{1 + 2\frac{1-c}{c} + 2\frac{\sqrt(1-c)}{c}\,\mathrm{cth}(z)}$$

$$J(z_\star) = d_M,\ 0 < z_\star < z_{max}$$

$$\frac{\Delta(z)}{a(z)} = 1 - 2\frac{\sqrt(1-c)}{c}\frac{J(z)z}{\mathrm{sh}^2(z)}\ ,\ \varphi(z) = \frac{J(z_\star)}{J(z)}z$$

$$0 < \frac{\Delta(z)}{a(z)} < \frac{\Delta(z_\star)}{a(z_\star)} < 1$$